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Markov Chains for Random Urinalysis I: Age-Test Model

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Since any drug abuse impacts readiness, health, and safety, continuing evaluation and improvement of the Navy's program is required. One method of improving the Navy's program is to develop and analyze alternative testing strategies.

The Nuclear Regulatory Commission (NRC) proposed a urinalysis testing strategy based on time since last test. That is, the probability of a person being tested depends on the amount of time since the person was last tested. Southern California Edison (SCE) has implemented a variation of the NRC proposal. Urinalysis testing strategies based on time since last test are defined by a high testing rate for personnel not yet tested in a given time period and a low testing rate for previously tested personnel with negative results in a given time period.

The probability distribution of random urinalysis tests is modeled under a general class of age-test urinalysis strategies. Age-test is a particular Markov chain with the probability of being tested defined as a function of time since last test. The NRC proposal, the SCE program, and current Navy practice can all be modeled as age-test Markov chains. Various age-test strategies are analyzed.

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Foreword

This report was prepared as part of the Markov Models of Random Urinalysis Sampling Procedures project (work unit 92PODD911) and the Statistical Methods for Drug Testing project (program element 0305889N), under the sponsorship of the Chief of Naval Personnel (PERS-63). The objectives of these projects include effectiveness of using urinalysis strategies based on time since last test to improve the Navy's drug deterrence program and development of a unified set of statistical methodologies for the analysis of drug testing programs and data. The work described here was performed during FY92 and FY93. Related work includes *Probability of Detection of Drug Users by Random Urinalysis in the U.S. Navy* (NPRDC-TN-93-2).

The opinions expressed in this paper are those of the authors, are not official, and do not necessarily reflect the views of the Navy Department.

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MURRAY W. ROWE
Director, Manpower Systems Department

Summary

Background

Since any drug abuse impacts readiness, health, and safety, continuing evaluation and improvement of the Navy's program is required. One method of improving the Navy's program is to develop and analyze alternative testing strategies.

The Nuclear Regulatory Commission (NRC) proposed a urinalysis testing strategy based on *time since last test*. That is, the probability of a person being tested depends on the amount of time since the person was last tested. Southern California Edison (SCE) has implemented a variation of the NRC proposal. Urinalysis testing strategies based on time since last test are defined by a *high* testing rate for personnel not yet tested in a given time period and a *low* testing rate for previously tested personnel with negative results in a given time period.

Objective

The objective of this work is to determine if urinalysis strategies proposed by NRC and implemented by SCE could be used to improve the Navy's drug screening program.

Approach

The probability distribution of random urinalysis tests is modeled under a general class of age-test urinalysis strategies. Age-test is a particular Markov chain with the probability of being tested defined as a function of time since last test. The NRC proposal, the SCE program, and current Navy practice can all be modeled as age-test Markov chains. Various age-test strategies are analyzed.

Results

The age-test Markov chain was used to analyze five different urinalysis testing strategies: two NRC proposed alternatives, the SCE process, and two models of the Navy's program assuming a 15% monthly testing rate.

This age-test Markov chain has 13 states. The process is observed monthly and states 1 through 12 are defined by the number of months since an individual was last tested. State 13 includes individuals who were tested 13 or more months ago.

Age-test urinalysis strategies can be used to reduce the variance of the number of tests per person per year, while keeping the average number of tests unchanged. This allows a reduction in both the number of people who are not tested and the number of people tested multiple times. The system also becomes more predictable and, hence, potentially subject to gaming by drug users.

The five urinalysis strategies are summarized below.

Strategy	Annual Testing Rate (%)	Constant Monthly Rate	Probability Not Tested: Within 1 Year	Probability Not Tested: Within 1 Year Given Just Tested
NRC—A	103	No	.100	.738
NRC—B	300	Yes	.032	.032
SCE	130	No	.052	.478
Navy—A	180	Yes	.142	.142
Navy—B	180	No	.081	.108

Note. NRC = Nuclear Regulatory Commission, SCE = Southern California Edison.

These strategies have widely varying annual testing rates (103-300%), and either constant or varying monthly rates. The varying annual rates imply varying costs of the programs. A preferred strategy should have low annual testing rates to lower costs, a low probability of not being tested within 1 year, and a low probability of not being tested within 1 year given just tested. When this last probability is high, the system is subject to gaming by drug users.

Conclusions

Markov chains provide a framework for the systematic analysis of drug testing strategies based on time since last test. The steady state distribution provides estimates of the number of tests per month and the number of people who have not been tested in the past year. The distribution of the number of tests in a fixed time period (e.g., year), given any initial state, can be calculated. Furthermore, given test or cost estimates, the relative merits of different testing strategies can be easily calculated. In general, age-test urinalysis strategies trade off predictability for reduced *tail area*. Here we mean the tail area of the distribution of the number of tests in a fixed time period. Age-test strategies provide fewer people not tested within 1 year and fewer people tested excessively during 1 year. Age-test strategies are also more predictable, have lower variance in the number of tests, and as a result are subject to gaming by drug users. Future work will quantify these trade-offs.

Both the NRC alternative A and the SCE process have some undesirable properties. These strategies involve large differences in the testing rates between people tested within the past year and those who were not tested. This implies that once tested there is a high probability of not being tested again within 1 year. These probabilities are 0.74 for NRC alternative A and 0.48 for the SCE process. The SCE process is such that almost half the tests every month are given to people who know they will be tested. For these reasons we do not recommend either NRC alternative A or the SCE process.

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1.0 Introduction

1.1 Background

The U.S. Navy's *zero tolerance* drug policy has been in effect since 1981. Since then the Navy has pursued an aggressive urinalysis testing program. The objectives of this testing program have been to deter and detect drug abuse, as well as provide data on the prevalence of drug abuse. All officer and enlisted personnel are subject to random urinalysis testing on a continuing basis. Current policy (Chief of Naval Operations 1990), directs Navy commands to test 10 to 20% of their members each month. The Navy's random urinalysis program has been considered successful. The proportion of service members sampled testing positive for drugs fell from 7% to 1% between 1983 and 1991. However, since any drug abuse impacts readiness, health, and safety, continuing evaluation and improvement of the Navy's program is required. One method of improving the Navy's program is to develop and analyze alternative testing strategies.

The Nuclear Regulatory Commission (NRC) (1988) proposed a urinalysis testing strategy based on *time since last test*. That is, the probability of a person being tested depends on the amount of time since the person was last tested. Southern California Edison (SCE) has implemented a variation, (Murray & Talley 1988), of the NRC proposal at the San Onofre Nuclear Generating Station. Urinalysis testing strategies based on time since last test are defined by a *high* testing rate for personnel not yet tested in a given time period and a *low* testing rate for previously tested personnel with negative results in a given time period. This strategy may provide balance among program objectives, including detection, deterrence, a high probability of testing some fixed number of times, a low probability of testing more than some fixed number of times, cost effectiveness, ease of administration, and avoidance of discrimination. The NRC's (Nuclear Regulation Commission, 1989) adopted rules and regulations for urinalysis do not require a *time since last test* strategy. However, SCE continues to use their variation of this strategy with NRC approval.

The advantage of a *time since last test* strategy is shown in the following example. Assume the Navy's objectives at each command are: 90% of personnel should be tested at least once each year and personnel already tested within 1 year should be tested at a rate of 2.5% per month. A simple random sampling (with replacement) strategy that meets these objectives would require an average of 2.1 tests[1] per person per year. However, a strategy based on time since last test could meet these objectives with an average of 1.0 tests per person per year. An annual savings of 5.94 million (based on a decrease of 1.1 tests per person for 600,000 people at \$9 per test) could be realized by using an age-test strategy in this example.

Markov chains provide a framework for the systematic analysis of drug testing strategies based on time since last test. The current Navy simple random sampling strategy is included within this framework. A related use of Markov chains, modeling classes of drug users, is given in Evanovich (1985). Previous work (Thompson & Boyle, 1992), includes models of detection of drug users.

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[1]Based on a monthly rate τ, the probability of being tested at least once within 12 months is $1 - (1 - \tau)^{12} = 0.9$. Therefore $\tau = 0.175$ and the average annual number of tests is $12\tau = 2.1$.

1.2 Objective

The objective of this work is to determine if urinalysis strategies proposed by NRC and implemented by SCE could be used to improve the Navy's drug screening program.

1.3 Approach

The probability distribution of random urinalysis tests is modeled under a general class of *age-test* urinalysis strategies. Age-test is a particular Markov chain with the probability of being tested defined as a function of time since last test. The NRC proposal, the SCE program, and current Navy practice can all be modeled as age-test Markov chains with 13 states. In this case, states 1 through 12 correspond to 1 through 12 months since last test and state 13 is defined as over 12 months since last test. The probability of being tested, given the current state, defines an *age-test urinalysis strategy*. Various age-test strategies are analyzed.

2.0 Markov Chains

This section briefly develops the theory and notation that will be used in the remainder of the report. Chapters 1 and 2 of Hoel, Port, & Stone (1972) are the primary source for this material.

Consider a system that can be in any one of a finite number of states. This set of states is denoted Land is called the *state space* of the system. The system is observed at discrete points in time $n = 0, 1, 2, \ldots$, and X_n denotes the state of the system at time n. For our purposes X_n is a random variable; that is, the system is not deterministic.

Systems with the property that only the present state influences the future are called *Markov chains*. In such systems, knowledge of the path taken to reach the present state cannot help predict the future.The Markov chain is a simple generalization of systems of independent random variables.

The Markov property is defined by

$$P(X_{n+1} = x_{n+1} \mid X_0 = x_0, \ldots, X_n = x_n) = P(X_{n+1} = x_{n+1} \mid X_n = x_n).$$

The conditional probabilities $P(X_{n+1} = x_{n+1} \mid X_n = x_n)$ are called *transition probabilities*. When the transition probabilities are independent of time n, they are called *stationary* transition probabilities. The model developed in this paper has random variables that satisfy the Markov property and have stationary transition probabilities. We represent the transition probabilities by the matrix **P**, where

$$\mathbf{P}(x, y) = P(X_{k+1} = y \mid X_k = x).$$

The product of **P** with itself n times yields the n-step transition matrix where

$$\mathbf{P}^n(x, y) = P(X_{k+n} = y \mid X_k = x).$$

Given a stationary transition matrix $\mathbf{P}$ and an initial distribution Π_0, all probabilities associated with the chain are uniquely determined. In particular, the distribution of X_n is

$$\Pi_n = \Pi_0 \mathbf{P}^n.$$

Hitting times are random variables which play an important role in the theory of Markov chains. For $y \in \mathrm{L}$, the hitting time T_y is defined as the first positive time the chain resides at state y, i.e.,

$$T_y = \min\,(n \geq 1 : X_n = y).$$

Define

$$\rho_{xy} = P_x(T_y < \infty) = P(T_y < \infty \mid X_0 = x)$$

as the probability that a Markov chain starting at x will hit y in finite time. A state y is transient if $\rho_{yy} < 1$, or if starting at y there is some positive probability that the chain will never return to y. A state is recurrent if $\rho_{yy} = 1$. Furthermore, all recurrent states in a finite state Markov chain have the property that the mean return time m_y is positive and finite, where

$$m_y = E_y(T_y) = E(T_y \mid X_0 = y).$$

Such states are called positive recurrent. A fundamental theorem of finite Markov chains states that the class of positive recurrent states, L_p, is nonempty and is partitioned into closed irreducible subclasses. A class of states C is closed irreducible if, once in C, a chain cannot leave C and all states in C lead to each other with positive probability.

Associated with certain Markov chains are special distributions which satisfy

$$\sum_x \Pi(x)\,\mathbf{P}(x, y) = \Pi(y),\; y \in \mathrm{L}.$$

Such a distribution Π is called stationary and a chain with a stationary initial distribution will have the property that

$$P(X_n = y) = \Pi(y),\; y \in \mathrm{L}$$

for all $n \geq 0$. When a stationary distribution Π satisfies

$$\lim_{n \to \infty} \mathbf{P}^n(x,y) = \Pi(y),\; x, y \in \mathrm{L}$$

then Π is called a steady state distribution. This is a strong condition and implies that Π is the unique stationary distribution and

$$\lim_{n \to \infty} P(X_n = y) = \lim_{n \to \infty} \sum_x \Pi_0(x)\,\mathbf{P}^n(x, y) = \Pi(y),\; x, y \in \mathrm{L} \qquad (1)$$

for an arbitrary initial distribution Π_0. Equation 1 means that, regardless of the chain's initial distribution, for large n the distribution of X_n approximates the steady state distribution. It is important to know when stationary distributions exist and when a stationary distribution is steady state. Theorem 5 (Hoel et al., 1972, p. 64) states that an irreducible positive recurrent Markov chain has a unique stationary distribution given by

$$\Pi(y) = \frac{1}{m_y}, \quad y \in L \tag{2}$$

Theorem 7 (Hoel et al., 1972, p. 73) states that when such a chain is aperiodic the stationary distribution defined by equation 2 is steady state. A sufficient condition for aperiodicity is that $\mathbf{P}(x, x) > 0$ for at least one $x \in L$.

It is also necessary to define the concept of occupation times and their associated probabilities. Define the occupation time

$$N_n(y) = \sum_{m=1}^{n} 1_y(X_m) \tag{3}$$

as the number of times the chain is in state y in n time periods, where 1_y is the indicator function

$$1_y(z) = \begin{cases} 1 \text{ if } z = y \\ 0 \text{ if } z \neq y. \end{cases}$$

The occupation time probabilities are

$$P_x(N_n(y) = m) = P(N_n(y) = m \mid X_0 = x) \tag{4}$$

for $m = 0, 1, \ldots, n$. This is the probability of m visits to state y in n time periods given that the chain started in state x.

3.0 Age-Test Model

Using the previously described notation and theory, we now define a class of age-test Markov chains where the states are defined by time since last test. The transition matrix is

$$\mathbf{P} = \begin{bmatrix} p_1 & q_1 & 0 & 0 & \ldots & 0 \\ p_2 & 0 & q_2 & 0 & \ldots & 0 \\ p_3 & 0 & 0 & q_3 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ p_d & 0 & 0 & 0 & \ldots & q_d \\ p_{d+1} & 0 & 0 & 0 & \ldots & q_{d+1} \end{bmatrix}. \tag{5}$$

A similar model, called age-replacement, where $p_{d+1} = 1$ is presented in Taylor and Karlin (1984). Later we will be primarily interested in the case $d = 12$ and testing is conducted monthly. In general, an individual is in state i if last tested i time periods ago. An individual resides in state $d+1$ if last tested $d+1$ or more time periods ago. For each state an individual is either tested with probability p_i, in which case the individual moves to state 1, or the individual ages 1 time period with probability $q_i = 1 - p_i$. Hence, the chain is named age-test. For example, the Navy program with testing unrelated to time since last test and a 15% monthly testing rate would have all the p_i's equal to 0.15.

When the restrictions $0 < p_1 \ldots p_d < 1$ and $0 < p_{d+1} \leq 1$ are placed on the transition matrix in equation 5, all states lead to all other states with positive probability. Hence, the chain constitutes a single irreducible positive recurrent class and there is a unique stationary distribution. Let $\Pi = (\pi_1, \pi_2, \ldots \pi_{d+1})$. Solving $\Pi = \Pi P$ with $\Sigma\, \pi_k = 1$ will give the stationary distribution Π. Writing out the equations we have the following

$$
\begin{array}{rclcccccccc}
\pi_1 & = & p_1\pi_1 & + & p_2\pi_2 & + & \ldots & + & p_d\pi_d & + & p_{d+1}\pi_{d+1} \\
\pi_2 & = & q_1\pi_1 & & & & & & & & \\
\pi_3 & = & & & q_2\pi_2 & & & & & & \\
\vdots & & & & & & & & & & \\
\pi_{d+1} & = & & & & & & & q_d\pi_d & + & q_{d+1}\pi_{d+1} \\
1 & = & \pi_1 & + & \pi_2 & + & \ldots & + & \pi_d & + & \pi_{d+1}
\end{array}
$$

Solving in terms of π_1 we obtain

$$
\begin{array}{rcl}
\pi_1 & = & \pi_1 \\
\pi_2 & = & q_1\pi_1 \\
\pi_3 & = & q_2\pi_2 = q_2 q_1 \pi_1 \\
\vdots & & \\
\pi_k & = & q_{k-1}\pi_{k-1} = q_{k-1}q_{k-2}\cdots q_1\pi_1 \\
\vdots & & \\
\pi_d & = & q_{d-1}\pi_{d-1} = q_{d-1}q_{d-2}\cdots q_1\pi_1 \\
\pi_{d+1} & = & (q_d / p_{d+1})\pi_d = (q_d q_{d-1} \cdots q_1 / p_{d+1})\pi_1.
\end{array}
$$

Since $\Sigma\pi_k = 1$ then

$$\pi_1 = 1 / [1 + q_1 + q_1 q_2 + q_1 q_2 q_3 + \ldots + (q_1 q_2 \cdots q_{d-1}) + (q_1 q_2 \cdots q_d / p_{d+1})].$$

Since $p_1 > 0$, the chain is aperiodic and Π is steady state with, according to equation 2, $\pi_i = 1/m_i$. This is the reciprocal of the mean return time to state i.

In Section 4 we calculate the occupation time probabilities for state 1 using the age-test model. The occupation time probabilities for 12 time periods provide the distribution of the annual number of tests. A visit to state 1 is equivalent to being tested. Observe that from any initial state there are 2^{12} possible paths over 12 periods. The probability of each path is calculated and the results are aggregated to get the conditional probabilities in equation 4. The unconditional distribution of the number of tests in 12 time periods, $(N_{12}(1))$, with initial distribution equation 6 is determined by

$$P(N_{12}(1) = m) = \sum_{i=1}^{13} \pi_i P_i (N_{12}(1) = m).$$

We also observe the average annual number of tests is

$$\begin{aligned} E(N_{12}(1)) &= E\left(\sum_{m=1}^{12} 1_1 (X_m) \right) \\ &= \sum_{m=1}^{12} P(X_m = 1) \\ &= 12\, \pi_1 \end{aligned}$$

since Π is stationary.

Finally, note the probability of not ing tested within 1 year is

$$P(N_{12}(1) = 0) = \pi_{13}. \tag{7}$$

Specifically, the event $N_{12}(1) = 0$ is equivalent to the chain being in state 13 at time 12. Because the initial distribution is stationary, all distributions are stationary and $P(X_{12} = 13) = \pi_{13}$. See Sellke (1984) for further details.

4.0 Applications

The age-test Markov chain was used to analyze five different urinalysis testing strategies: two NRC proposed alternatives, the SCE process, and two models of the Navy's program assuming a 15% monthly testing rate.

This age-test Markov chain has 13 states. The process is observed monthly and states 1 through 12 are defined by the number of months since an individual was last tested. State 13 includes individuals who were tested 13 or more months ago. Age-test urinalysis strategies can be used to reduce the variance of the number of tests per person per year, while keeping the average number of tests unchanged. This allows a reduction in both the number of people who are not tested and the number of people tested multiple times. The system also becomes more *predictable* and, hence, potentially subject to gaming by drug users. The specific examples follow.

4.1 Nuclear Regulatory Commission Proposals (NRC)

Two urinalysis testing alternatives proposed by the NRC (1988) are analyzed in this section. Alternative A required that at least 90% of the individuals are tested each year and that testing rates for individuals already tested with negative results be at least 2 1/2% per month. The age-test model with

$$p_i \geq 0.025, i = 1, \ldots, 12$$

$$\pi_{13} \leq 0.1 \tag{8}$$

satisfies these requirements. Minimizing[2] π_1, the average number of tests per month per person, subject to equation 8 yields a solution with $p_1 = p_2 = \ldots = p_{12} = 0.025$ and $p_{13} = 0.6338$. See the Appendix for a detailed analysis. Results are summarized in Table 1 and Figure 1. The last column in the table and the figure were calculated using equation 4 and enumerating all possible transitions over 12 months. The advantages of this alternative include: (1) the average number of tests per person per year is 1.03, (2) 90% of the people are tested at least once per year, and (3) 12% of the people are tested more than once. The major disadvantage is that once tested, people have a 74% chance of not being tested for the next 12 months.

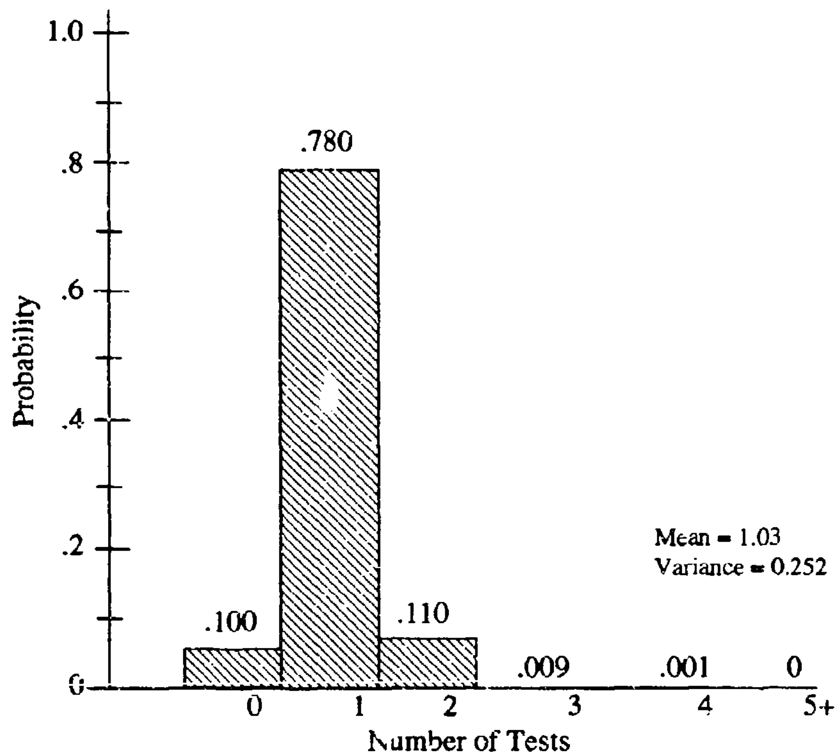
Table 1

Probabilities From Age-Test Model of NRC Alternative A

	Probability		
State	Tested This Month	Steady State	Not Tested Next 12 Months
1	.025	.086	.738
2	.025	.084	.277
3	.025	.082	.104
4	.025	.080	.039
5	.025	.078	.015
6	.025	.076	.006
7	.025	.074	.002
8	.025	.072	.001
9	.025	.070	.000
10	.025	.068	.000
11	.025	.067	.000
12	.025	.065	.000
13	.634	.100	.000

Note. NRC = Nuclear Regulatory Commission.

[2]This and subsequent optimization problems were solved using the Microsoft Excel Solver, Microsoft Corporation (1991), on an IBM compatible personal computer.



Note. NRC = Nuclear Regulation Commission.

Figure 1. Steady state distribution of number of tests within 12 months for age-test model of NRC alternative A.

Alternative B required that tests must be administered throughout the year at an annual rate equivalent to 300% of the population. The age-test model with

$$p_i > 0, \quad i = 1, \ldots, 13$$

$$\pi_1 = 300\% / 12 \text{ months} = 0.25 \qquad (9)$$

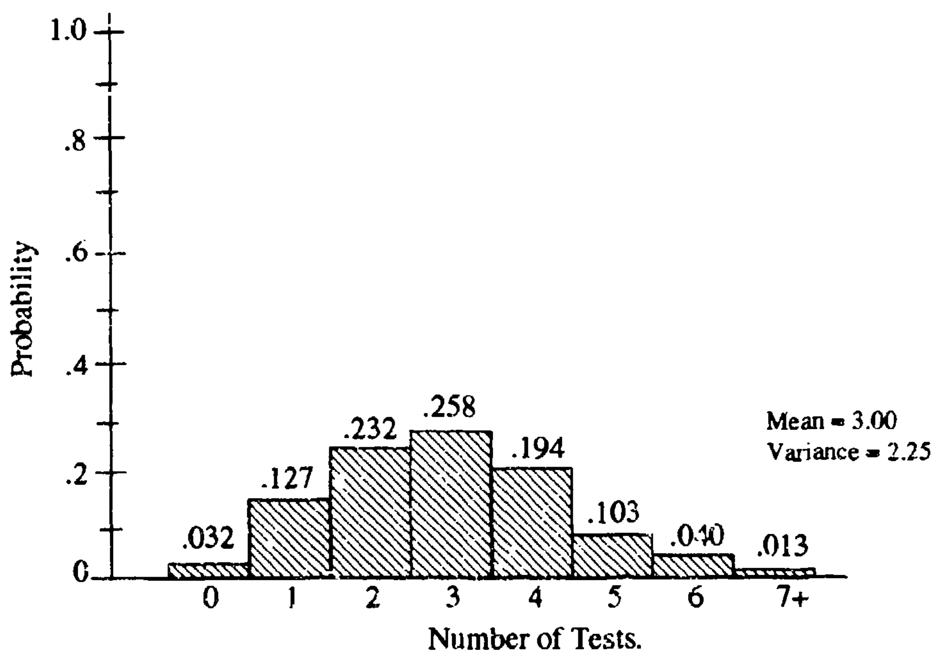
meets these requirements. Letting $p_1 = p_2 = \ldots = p_{13} = 0.25$ satisfies equation 9. Results are summarized in Table 2 and Figure 2. The advantages of this alternative include: 97% of the people are tested at least once and the current state provides no information on the future of the process. Any strategy with all the p_i equal has the latter advantage. When the p_i's are equal, the distribution of the number of tests in 1 year is simply the binomial. A disadvantage of alternative B is the high cost of three tests per person per year.

Table 2

Probabilities From Age-Test Model of NRC Alternative B

State	Probability: Tested This Month	Probability: Steady State	Probability: Not tested Next 12 Months
1	.25	.250	.032
2	.25	.188	.032
3	.25	.141	.032
4	.25	.105	.032
5	.25	.079	.032
6	.25	.059	.032
7	.25	.044	.032
8	.25	.033	.032
9	.25	.025	.032
10	.25	.019	.032
11	.25	.014	.032
12	.25	.011	.032
13	.25	.032	.032

Note. NRC – Nuclear Regulatory Commission.



Note. NRC – Nuclear Regulatory Commission.

Figure 2. Steady state distribution of number of tests within 12 months for age-test model of NRC alternative B.

4.2 Southern California Edison (SCE)

SCE has implemented a *composite random sampling* (Murray & Talley, 1988), approach to urinalysis. Their approach is based on a sampling scheme that is part sampling with replacement and part sampling without replacement. The entire population is sampled at a specified rate with replacement. People who have not been sampled within the past year are sampled at another specified rate without replacement. SCE states a 5% annual chance of not being tested and a 130% average annual testing rate. The process in use at SCE can be modeled as an age-test Markov chain. An age-test model with

$$\begin{aligned} \pi_1 &\leq 130\% \,/\, 12 \text{ months} = 0.1083 \\ \pi_{13} &\leq 0.05 \qquad (10) \\ p_1 &= p_2 = \ldots = p_{12} \end{aligned}$$

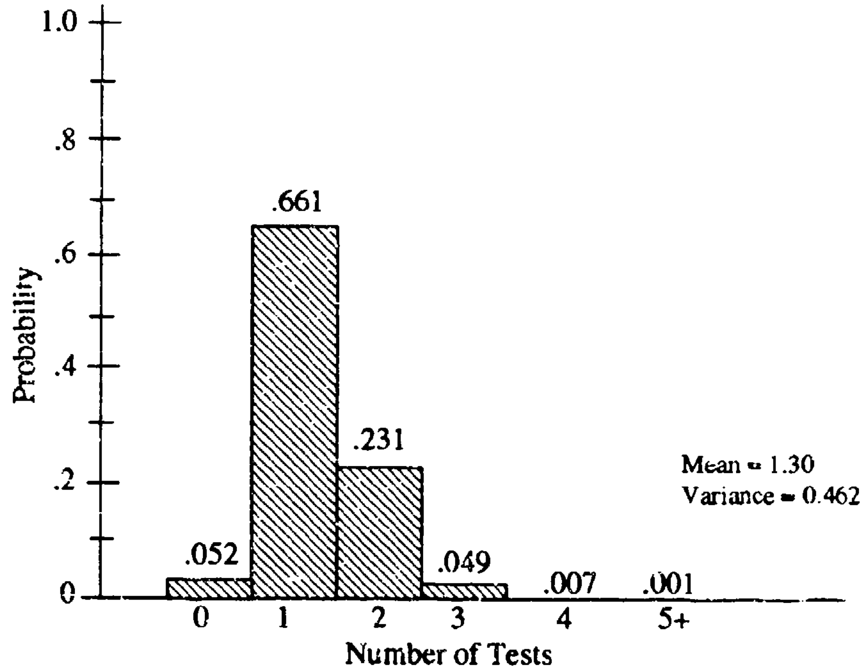
meets the specifications of the SCE sampling scheme. There is no feasible solution to equation 10. The following two optimizations were performed to find an age-test strategy as close to meeting equation 10 as possible. Recall that π_1, the steady state probability for state 1, represents the average monthly testing rate, and π_{13}, the steady state probability for state 13, represents the average annual **not** tested rate. Minimizing π_1 subject to the restrictions on p_i and π_{13} in equation 10 yields $p_1 = p_2 = \ldots = p_{12} = 0.0645$, $p_{13} = 1.0$ and $\pi_1 = 0.1113$. Minimizing π_{13} subject to the restrictions on p_i and π_1 in equation 10 yields $p_1 = p_2 = \ldots = p_{12} = 0.0596$, $p_{13} = 1.0$ and $\pi_{13} = 0.0518$. The solutions from both of these optimizations are close to the results given by SCE (Murray & Talley, 1988). Results from the latter optimization are summarized in Table 3 and Figure 3. A problem with this process is that every month almost one half ($0.052/0.108 = 0.48$) of the tests are given to people who are in state 13 and, therefore, know they are being tested that month.

Table 3

Probabilities From Age-Test Model of SCE's Process

	Probability		
State	Tested This Month	Steady State	Not Tested Next 12 Months
1	.06	.108	.478
2	.06	.102	0
3	[illegible]	.096	0
4	.06	.090	0
5	.06	.085	0
6	.06	.080	0
7	[illegible]	.075	0
8	.06	.070	0
9	.06	.066	0
10	.06	.062	0
11	.06	.059	0
12	.06	.055	0
13	1	.052	0

Note. SCE = Southern California Edison.



Note. SCE = Southern California Edison.

Figure 3. Steady state distribution of number of tests within 12 months for age-test model of SCE's process.

4.3 United States Navy (U.S. Navy)

U.S. Navy policy, Chief of Naval Operations (1990), directs commands to test 10 to 20% of their personnel each month. Age-test models with

$$0.10 \leq p_i \leq 0.20, \quad i = 1, \ldots, 13 \tag{11}$$

meet this requirement[3]. Two different strategies consistent with equation 11 are presented here. In the first strategy (Navy-A), let $p_1 = \ldots = p_{13} = 0.15$. Choosing the p_i all equal is consistent with current Navy practice. In this model testing is independent of time since last test. The value 0.15 was chosen because it is the midpoint between 0.10 and 0.20. Results are summarized in Table 4 and Figure 4.

[3]Strictly speaking $0.15 \leq \pi_1 \leq 0.20$ is sufficient to meet this requirement.

Table 4

Probabilities From Age-Test Model of Navy Program At 15% Monthly Testing Rate (Navy-A)

	Probability		
State	Tested This Month	Steady State	Not Tested Next 12 Months
1	.15	.150	.142
2	.15	.127	.142
3	.15	.108	.142
4	.15	.092	.142
5	.15	.078	.142
6	.15	.067	.142
7	.15	.057	.142
8	.15	.048	.142
9	.15	.041	.142
10	.15	.035	.142
11	.15	.030	.142
12	.15	.025	.142
13	.15	.142	.142

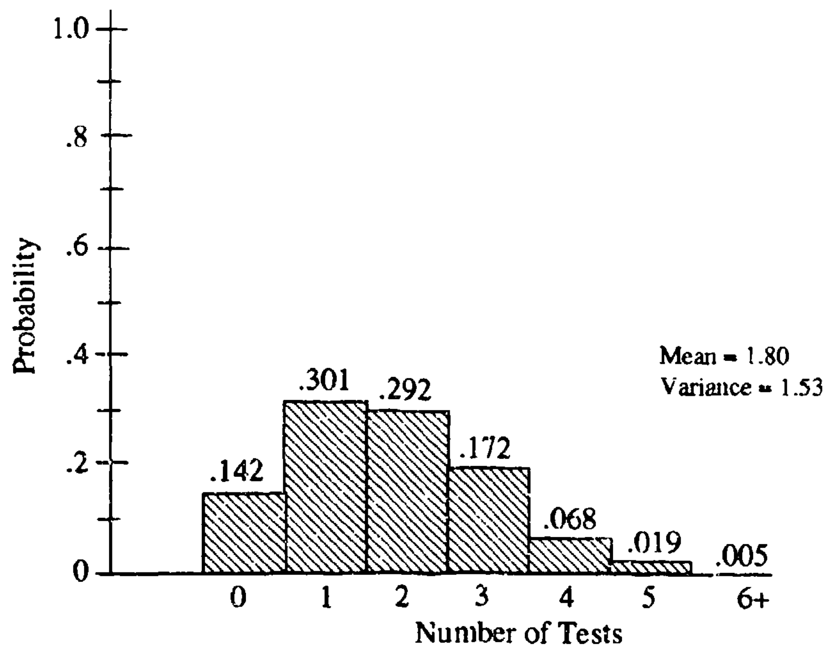


Figure 4. Steady state distribution of number of tests within 12 months for age-test model of Navy program at 15% monthly testing rate (Navy-A).

Recall π_1, the steady state probability of being in state 1, is the average monthly testing rate. A second strategy (Navy-B) with π_1 equal to Navy-A is presented for comparison purposes. The following minimization problem was solved: minimize π_{13} subject to $\pi_1 = 0.15$ and equation 11. Note π_{13} is the probability of not being tested within 1 year. Therefore, this strategy has the same average monthly testing rate and minimizes the probability of not being tested within 1 year. The solution is $\{p_1 = p_2 = p_3 = 0.1, p_4 = 0.1145, p_5 = \ldots = p_{13} = 0.2\}$. Results are summarized in Table 5 and Figure 5. In this case $\pi_{13} = .81$ as compared to $\pi_{13} = .142$ for Navy-A. Therefore, with an equal number of monthly tests and keeping the monthly testing rate between 10 and 20%, strategy Navy-B reduces the number of people not tested in a given year by 43% ([.142 - .081] / .142 x 100).

Table 5

Probabilities From Age-Test Model of Navy Program at 180% Annual Testing Rate

	Probability		
State	Tested This Month	Steady State	Not Tested Next 12 Months
1	.10	.150	.108
2	.10	.135	.096
3	.10	.122	.086
4	.11	.109	.076
5	.20	.097	.069
6	.20	.077	.069
7	.20	.062	.069
8	.20	.050	.069
9	.20	.040	.069
10	.20	.032	.069
11	.20	.025	.069
12	.20	.020	.069
13	.20	.081	.069

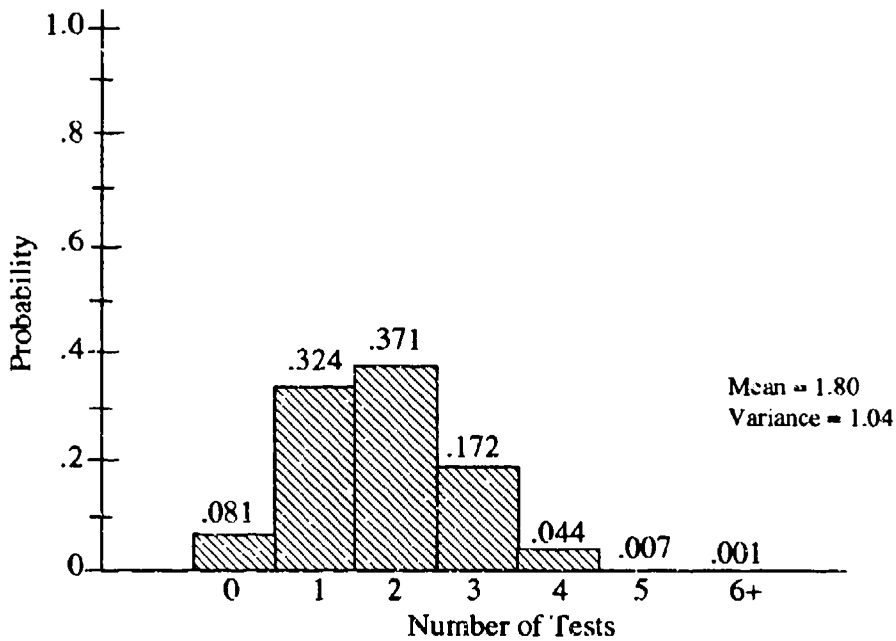


Figure 5. Steady state distribution of number of tests within 12 months for age-test model of Navy program at 180% annual testing rate (Navy-B).

The five urinalysis strategies presented in this section are summarized in Table 6. These strategies have widely varying annual testing rates (103 to 300%). This, of course, implies widely varying costs of the programs. A preferred strategy should have low annual testing rates to lower costs, a low probability of not being tested within 1 year, and a low probability of not being tested within 1 year given just tested. When this last probability is high, the system is subject to gaming by drug users.

Table 6

Summary of Age-Test Markov Chain Analysis of Five Urinalysis Strategies

Strategy	Annual Testing Rate (%)	Probability Not Tested	
		Within 1 Year	Within 1 Year Given Just Tested
NRC-A	103	.100	.738
NRC-B	300	.032	.032
SCE	130	.052	.478
Navy-A	180	.142	.142
Navy-B	180	.081	.108

5.0 Conclusions

Markov chains provide a framework for the systematic analysis of drug testing strategies based on time since last test. Under mild conditions on the transition probabilities, these Markov chains converge to steady state. The steady state distribution provides estimates of the number of tests per month and the number of people who have not been tested in the past year. Since this steady state solution can be expressed in closed form, optimization problems involving steady state can be formulated and solved.The distribution of the number of tests in a fixed time period (e.g., year), given any initial state, can be calculated.Furthermore, given test or cost estimates, the relative merits of different testing strategies can be easily calculated.

In general, age-test urinalysis strategies trade off predictability for reduced *tail area.* Here we mean the tail area of the distribution of the number of tests in a fixed time period. Age-test strategies provide fewer people not tested within 1 year and fewer people tested excessively during 1 year. Age-test strategies are also more predictable, have lower variance in the number of tests, and as a result are subject to gaming by drug users.

Both the NRC alternative A and the SCE process have some undesirable properties.These strategies involve large differences in the testing rates between people tested within the past year and those who were not tested. This implies that once tested there is a high probability of not being tested again within 1 year. These probabilities are 0.74 for NRC alternative A and 0.48 for the SCE process.The SCE process is such that almost one half the tests every month are given to people who know they will be tested.

Currently, extensions to the age-test model are under development.These include drug usage patterns and their impact on the probability of detection.This will help quantify the trade-off between predictability and the reduced tail area in the distribution of the number of tests mentioned above.

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Appendix

Derivation of Optimal Conditions

Derivation of Optimal Conditions

This appendix develops the solution to the following problem:

$$\text{Minimize } \pi_1 = \frac{1}{1 + q_1 + q_1q_2 + \ldots + q_1q_2 \cdots q_{d-1} + (q_1q_2 \cdots q_{d-1}q_d)/p_{d+1}} \tag{A1}$$

$$\text{Subject to } \pi_{d+1} = \frac{q_1q_2 \cdots q_{d-1}\, q_d}{p_{d+1}}\, \pi_1 \le \alpha$$

$$p_i \ge \beta; \quad i = 1, 2, \ldots, d$$

where $0 < \alpha, \beta < 1$. Clearly, the problem is equivalent to

$$\text{Minimize } 1 + q_1 + q_1q_2 + \ldots + q_1q_2 \cdots q_{d-1} + \frac{q_1q_2 \cdots q_{d-1}q_d}{p_{d+1}} \tag{A2}$$

$$\text{subject to } \frac{q_1q_2 \cdots q_{d-1}q_d}{p_{d+1}}\, \pi_1 \le \alpha \tag{A3}$$

$$p_i \ge \beta; \quad i = 1, 2, \ldots, d.$$

For given values of q_d and p_{d+1}, the objective function (A2) is largest at $q_1 = q_2 = \ldots = q_{d-1} = 1 - \beta$. Hence, we wish to solve:

$$\text{Maximize } \quad x = \frac{q_d}{p_{d+1}} \tag{A4}$$

$$\text{Subject to } \quad \frac{x}{[1/(1-\beta)^{d-1}]\,[(1-(1-\beta)^d)/\beta] + x} \le \alpha \tag{A5}$$

$$p_d \ge \beta$$

where (A5) is the result of manipulating (A3) and noting that $1 + (1-\beta) + \ldots + (1-\beta)^{d-1} = [1-(1-\beta)^d]/\beta$. Finally, solving (A5) for x we have the equivalence

$$\text{Maximize } \quad x = \frac{q_d}{p_{d+1}}$$

$$\text{Subject to } \quad x \le \frac{\alpha}{(1-\alpha)(1-\beta)^{d-1}} \; \frac{1-(1-\beta)^d}{\beta} = y(\alpha, \beta)$$

$$p_d \ge \beta.$$

Choosing $x = y(\alpha, \beta)$ with $p_d \geq \beta$ yields a solution. This reduces to

$$p_{d+1} = \frac{1}{y(\alpha, \beta)} - \frac{p_d}{y(\alpha, \beta)}, \quad p_d \geq \beta$$

and (p_d, p_{d+1}) must lie on a line. Figure A-1 illustrates the situation. When $(\alpha, \beta) = (\alpha_1, \beta_1)$, there are multiple solutions lying on L_1 to the right of the vertical line $p_d = \beta$. In such a configuration a unique solution can always be obtained by choosing the intersection of L_1 and the 45° line $L : p_d = p_{d+1}$. The case $(\alpha, \beta) = (\alpha_2, \beta_2)$ implies $p_{d+1} < p_d$. In this configuration a unique solution can be obtained by letting $p_d = \beta$ and taking p_{d+1} on L_2 at $p_d = \beta$ or $p_{d+1} = (1 - \beta)/ y(\alpha, \beta)$.

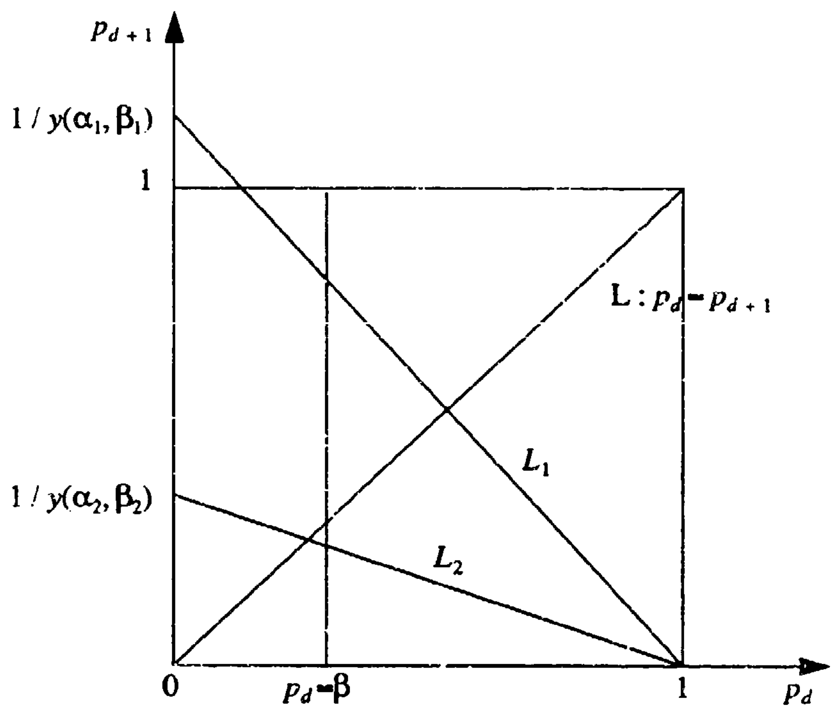

Figure A-1. Geometric interpretation of optimal region.

For any optimal solution, the minimum value of π_1 is obtained by substituting $y(\alpha, \beta)$ for q_d / p_{d+1} in (A1) and setting $q_1 = q_2 = \ldots = q_{d-1} = 1 - \beta$. This yields

$$\begin{aligned}\pi_1(\alpha, \beta) &= \frac{1}{1 + (1 - \beta) + \ldots + (1 - \beta)^{d-1} + (1 - \beta)^{d-1} y(\alpha, \beta)} \\ &= \frac{\beta(1 - \alpha)}{1 - (1 - \beta)^d} .\end{aligned}$$

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